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# THE INTERRELATION OF SMALL BODIES OF THE SOLAR SYSTEM

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# THE INTERRELATION OF SMALL BODIES OF THE SOLAR SYSTEM

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ABSTRACT. The author investigates the distribution of small bodies (asteroids, meteorites, comets and meteoric particles) by the magnitude of the constant (in the first approximation, $h_0$) of the Jacobi integral in the limited circular three-body problem. Conclusions: 1. Asteroids, meteorites and comets form, on the curve $N=f(h_0)$, pronounced independent groups in a rather narrow range $h_0$, while the distribution of meteoric particles is more uniform over a very wide range $h_0$; 2. Meteoric particles may be associated with all the above-mentioned small bodies, but primarily with short-period comets; 3. A complex and profound interrelation is found among systems of small bodies.

As is known, cosmogony of the solar system is related in the closest way /11* with the problem of the evolution of small bodies. Attempts have been made repeatedly to approach the investigation of this exceedingly complicated problem from the viewpoint of celestial mechanics. Thus, it has been necessary to ignore the effect of various physical factors obviously having definite value.

We are going to consider the asteroids, meteorites, comets and meteoric particles investigated here as small bodies and agree to consider any of these by the term "small body."

Celestial mechanics examines the limited circular three-body problem. A small body moves under the influence of the forces of gravitational attraction of two bodies: the Sun and Jupiter. The small body is taken as a mass point of infinite mass and is not attracted, but attracts. The Sun and Jupiter, having finite masses, describe circular orbits about a mutual center of inertia.

Even in this approximate mechanical system only one integral of the equations of motion--the Jacobi integral--is known.

As a basic criterion in the study of the systems of small bodies enumerated above, we chose the value of the constant Jacobi integral. From this standpoint our goal was to continue the work of A. N. Chibisova (ref. 1), T. V. Vodop'yanova (ref. 2), as well as their predecessors C. Charlier (ref. 3), A. Klose (ref. 4), and others. In contrast to preceding authors, we were interested not so much in the structure of the individual small-body systems as in the general interrelation of the small bodies.

*Numbers in the margin indicate original pagination in the foreign text.

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Thus, let x, y, z be the linear rectangular coordinates of the small body in the rotating system having its origin in the center of the mass of the Sun S, the plane xSy corresponding to the plane of the orbit of Jupiter I and the axis Sx passing through the center of mass I.

The equations of motion of the small body in the limited circular problem of three bodies will be /12

$$\ddot{x} = 2n_j\dot{y} + n_j^2 x - k^2\frac{x}{R^3} - k^2 m_j\frac{x-a_j}{\varrho^3} - n_j^2\frac{m_j}{1+m_j}a_j;$$

$$\ddot{y} = -2n_j\dot{x} + n_j^2 y - k^2\frac{y}{R^3} - k^2 m_j\frac{y}{\varrho^3};$$

$$\ddot{z} = -k^2\frac{z}{R^3} - k^2 m_j\frac{z}{\varrho^3}.$$

Here R and $\rho$ are the distances of the small body from the Sun and from Jupiter respectively; $m_j$ is the mass of Jupiter; the mass of the Sun is taken as unity; $\alpha_j$ is the mean distance from Jupiter to the Sun; $n_j$ is the angular velocity of the motion of Jupiter in an orbit assumed constant; $k^2$ is the constant of attraction. The Jacobi integral will have the form

$$\dot{x}^2 + \dot{y}^2 + \dot{z}^2 = v^2 = 2(U + h),$$

where v is the velocity of the small body in the system of rotating axes Sxyz; U is the force function and h is the Jacobi constant. Hence, h, expressed by the elements of the osculating Keplerian orbit of the small body, is equal to

$$h = -\left[k^2\left(\frac{1}{2a} + \frac{\sqrt{1+m_j}}{a_j\sqrt{a_j}}\sqrt{a(1-e^2)}\cos i'\right) + \frac{k^2 m_j}{\varrho} + \right.$$
$$\left. + \frac{n_j^2 m_j^2}{2(1+m_j)^2}a_j^2 - \frac{n_j^2 m_j}{1+m_j}a_j x\right],$$

where $\alpha$, e are the semimajor axes and eccentricity of the orbit of the small body; i' is the inclination of the orbital plane of the small body to the orbital plane of Jupiter. The mean motion of Jupiter in orbit is

$$n_j = \frac{k\sqrt{1+m_j}}{a_j\sqrt{a_j}}.$$

Proceeding as in reference 1, we represent h in the form of the sum of two terms--of the basic $h_0$ and correction $\delta h$.

$$h = h_0 + \delta h,$$

$$h_0 = -k^2\left(\frac{1}{2a} + \frac{\sqrt{1+m_j}}{a_j\sqrt{a_j}}\sqrt{a(1-e^2)}\cos i'\right),$$

$$\delta h = -\left(\frac{k^2 m_j}{\varrho} + \frac{h_j^2 m_j^2}{2(1+m_j)^2}a_j^2 - \frac{h_j^2 m_j}{1+m_j}a_j x\right).$$

The correction $\delta h$ can be ignored, because of which the error $\frac{\delta h}{h}$ in the worst case will be 2-3 percent. Therefore we calculated only the quantity $h_0$, which is called the Jacobi constant in the first approximation. Assuming here $i'=i$ (i.e., considering the coinciding planes of the orbit of Jupiter and of the ecliptic), which gives an error in $h_0$ generally about 1 percent. As a consequence of inaccuracy in the determination of the meteor orbit (by photographic observations) of about 2 percent, the error in $h_0$ for the comet orbit is of course, less. /13

The values of the constants in $h_0$ are taken as:

$k^2=0.000295912$; $m_j=0.00095479$; $\alpha_j=5.2028$.

We calculated the values of the Jacobi constants $h_0$ for the following small bodies: 1) 200 orbits of the meteor particles according to the well-known photographic observations from Harvard Observatory, while data were selected uniformly during the year with no preference whatsoever for continuous or sporadic material; 2) 150 "parabolic" comets which obviously must be considered as strongly prolate ellipses; 3) the orbits of 66 meteorites.

It is well-known that until recent years there have been no basic photographic observations of the trajectories of meteorites with the exception of one unique case (the meteorite rain of Prshibram in Czechoslovakia). According to available visual data, the initial velocity of meteorites is not always reliable or is entirely lacking. Therefore, in his time Nissl computed 36 meteorite orbits in the assumption of three values for the semimajor axis: $\alpha=2.0$ (ellipse); $\alpha=+\infty$ (parabola ); $\alpha=-0.5$ (hyperbola). I. S. Astapovich later similarly calculated the orbits for 66 meteorites. He published some of them in references 5 and 6. We used his calculations for $\alpha=2.0$ and $\alpha=+\infty$, since obviously the overwhelming majority of the real orbits is spread within these limits. Although in this manner we of course do not obtain a precise distribution of $h_0$ values for meteorites, this nevertheless gives some idea of their position in a series of small bodies on the curve $N=f(h_0)$ (see figure) within the prescribed limits.

For 70 short-period comets, values of $h_0$ from reference 2 were used, while for each of these comets they are taken as the mean from the values (undergoing oscillations totaling ±1, ±2 units of the seventh sign) for several of its appearances.

For asteroids the $h_0$ values were taken from reference 1.

The distribution of $N=f(h_0)$ for asteroids, meteorites, comets and meteoric particles has been established graphically in the figure. Here, along the abscissa the $h_0$ values are presented in units of $h_0 \cdot 10^7$, while on the ordinate, N is the relative frequency in percent. Only one percent of the orbits enter the region of $h_0$ values from $-2700 \cdot 10^7$ to $-2520 \cdot 10^7$, while in the interval from $-2520 \cdot 10^7$ to $-1980 \cdot 10^7$ there are generally no orbits whatsoever, because the axis of the abscissa from $h_0 < -1980 \cdot 10^7$ is not indicated in the figure.

Investigation of the systems of small bodies from the viewpoint of statistics of the Jacobi constants permits us to make the following conclusions:

1. Asteroids, comets and meteorites form sharply expressed independent groups concentrated in a rather narrow region $h_0$ when meteoric particles are highly dispersed, revealing a weak maximum corresponding to the maximum of short-period comets.

2. If it is permissible to interpret a given interrelation from the viewpoint of the formation of meteoric particles from other small bodies, there exist not one but several sources for the formation of these particles, although the chief role must obviously belong to the short-period comets.

3. Some portion of the asteroids and of the short-period comets forms a transition group of cometoids.

There also exists a region where we may speak of the possible affinity of several asteroids, meteorites, short-period comets and meteoric particles simultaneously. From this point of view it is necessary to recall a fact well-known /15
from meteor astronomy (ref. 5), that several orbits of comets, of meteor clusters and of meteorites intersect at one point in space and that it is quite possible to speak of some type of affinity for all these bodies. The present existence of a given system leads to the thought that it must have been found relatively recently.

4. Comets of groups (a) and (b) (see figure) basically do not reveal a relation (with the exclusion of individual cases). While somewhat dominant the proposition of the formation of comets of group (a) by means of the capture from group (b) has not been corroborated although individual examples are possible.

5. Ignoring the effect of physical factors (the Poynting-Robertson effect, corpuscular radiation, the interplanetary medium, etc), we allow ourselves the report of their important value for the evolution of meteoric material.

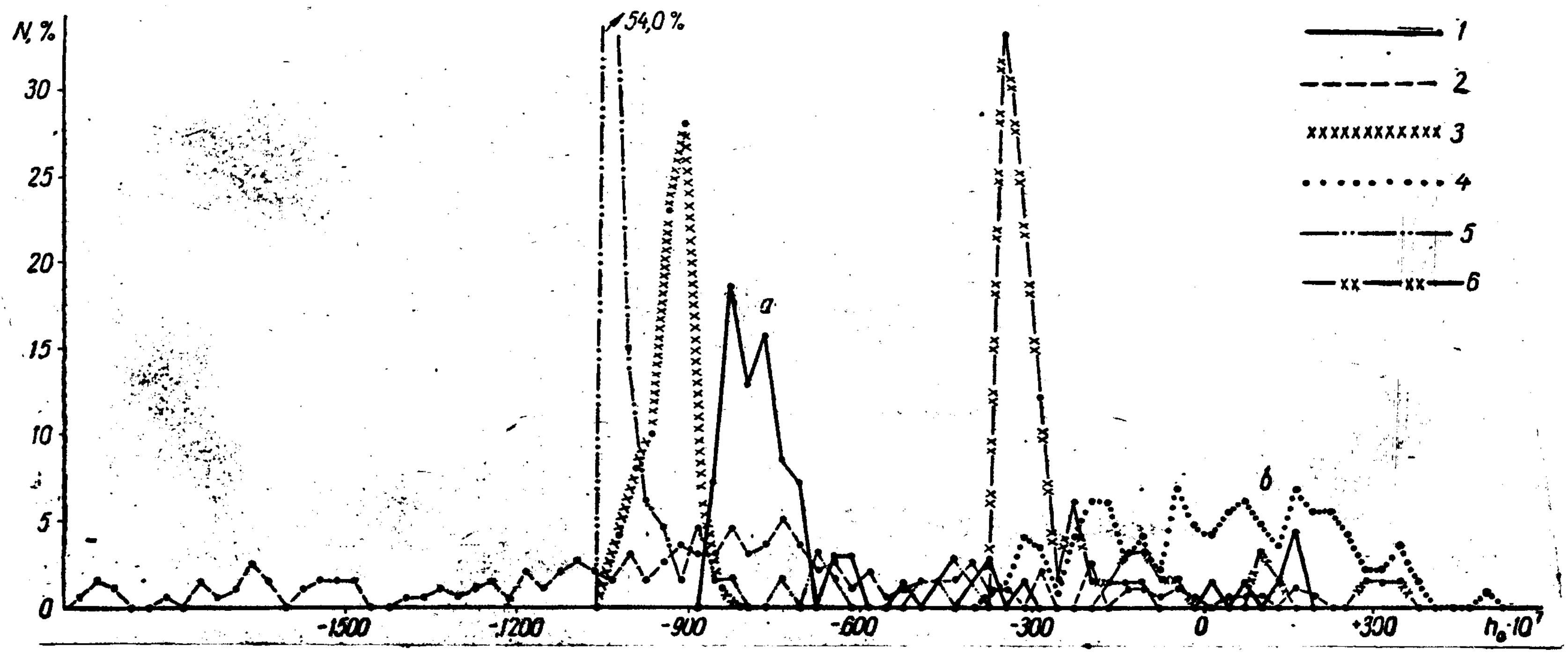


Figure. Distribution of Small Bodies by the Jacobi Constant.

1. Short-period comets (a);
2. Meteor particles;
3. Asteroids;
4. "Parabolic" comets (c);
5. Meteorites ($\alpha$=2.0);
6. Meteorites ($\alpha$=+$\infty$).

However, this basically relates to its more scattered component. The left part of the curve $N=f(h_0)$ for $h_0$ from $-2700\cdot10^7$ to $-1200\cdot10^7$ corresponding to meteor particles (a total of 22 percent) with $\alpha\leq1$, possibly is an illustration of what has been said; it is probably that the consideration of physical factors would displace these particles from the region $h_0<-1200\cdot10^7$ to the region $h_0>-1200\cdot10^7$.

In conclusion we note that only recently there appeared in print a work by M. Ovendena and A. Roy (ref. 7) concerning the nonuniformity for rather large time intervals of the use of the Jacobi integral in the limited circular problem of three bodies. This circumstance need not be significant in the investigation of comet-meteor systems, considering the relatively rapid growth of their evolution.

## REFERENCES

1. Chibisov, A. N. Trudy Gosudarstvennogo Astronomicheskogo Instituta imeni P. K. Shternberga (GAISh), Vol. 9, No. 1, 1936.
2. Vodop'yanova, T. V. Trudy GAISh, Vol. 9, No.2, 1939 (1940).
3. Charlier, C. Die Mechanik des Himmels (Celestial Mechanics). Vol. 2, p. 289.
4. Klose, A. Vierteljahrsschr. der Astr. Gesellsch., Vol. 63, 1928.
5. Astapovich, I. S. Astronomicheskiy Zhurnal, Vol. 16, No. 6, 1939.
6. --- Priroda, No. 1, 1940.
7. Ovenden, M. W. and Roy, A. E. M. N., Vol. 123, No. 1, 1961.

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